

NONLINEAR DIAGNOSTICS USING AC DIPOLES *

S. Peggs, BNL, Upton, NY

There are three goals in the accurate nonlinear diagnosis of a storage ring. First, the beam must be moved to amplitudes many times the natural beam size. Second, strong and long lasting signals must be generated. Third, the measurement technique should be non-destructive.

Conventionally, a single turn kick moves the beam to large amplitudes, and turn-by-turn data are recorded from multiple beam position monitors (BPMs) [1-6]. Unfortunately, tune spread across the beam causes the center of charge beam signal to “decohere” on a time scale often less than 100 turns. Filamentation also permanently destroys the beam emittance (in a hadron ring). Thus, the “strong single turn kick” technique successfully achieves only one out of the three goals. AC dipole techniques can achieve all three. Adiabatically excited AC dipoles slowly move the beam out to large amplitudes. The coherent signals then recorded last arbitrarily long. The beam maintains its original emittance if the AC dipoles are also turned off adiabatically, ready for further use.

The AGS already uses an *RF dipole* to accelerate polarized proton beams through depolarizing resonances with minimal polarization loss [7]. Similar AC dipoles will be installed in the horizontal and vertical planes of both rings in RHIC [8]. The RHIC AC dipoles will also be used as spin flippers, and to measure linear optical functions [9].

1 LINEAR MOTION

Horizontal motion is described using complex phasors

$$z \equiv x' + ix = a e^{i\phi} \quad (1)$$

so that the unperturbed one turn motion is just

$$z_{t+1} = R z_t \quad (2)$$

where $R = \exp(i 2\pi Q_X)$. Here Q_X is the betatron tune, and the normalized coordinates x and x' both have the dimensions of length. An AC dipole just after the reference point gives a real normalized angular kick on turn t of

$$\Delta z_t = \Delta x' = \delta \cos(2\pi Q_D t + \psi_0) \quad (3)$$

where Q_D is the drive tune and ψ_0 is the initial phase. The AC dipole strength is $\delta = (BL/(B\rho)) \beta_D$, where BL is the integrated field amplitude, $(B\rho)$ is the rigidity, and β_D is the Twiss function at the dipole.

If $z = z_0$ just before the first dipole kick, then the net displacement phasor on turn T is

$$z_T = R^T z_0 + (R^T \Delta z_0 + R^{T-1} \Delta z_1 \cdots + R^1 \Delta z_{T-1}) \quad (4)$$

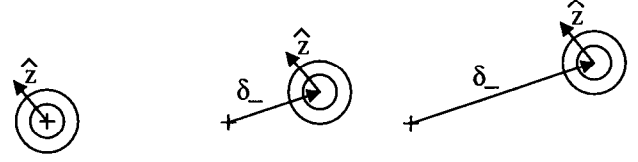


Figure 1: Adiabatic excitation of an AC dipole, in the rotating frame. The circles represent single particle motion.

The *exact* general solution for linear motion is [10]

$$z_T = \hat{z} e^{i 2\pi Q_X T} + \delta_- e^{i 2\pi Q_D T} - \delta_+ e^{-i 2\pi Q_D T} \quad (5)$$

where $\hat{z} = z_0 - \delta_- + \delta_+$ is a constant given by the initial conditions, and the complex AC dipole strengths are

$$\begin{aligned} \delta_- &= \frac{\delta}{4} \frac{\exp(-i[\pi Q_- - \psi_0])}{\sin(\pi Q_-)} \\ \delta_+ &= \frac{\delta}{4} \frac{\exp(i[\pi Q_+ - \psi_0])}{\sin(\pi Q_+)} \end{aligned} \quad (6)$$

where $Q_- = Q_D - Q_X$ and $Q_+ = Q_D + Q_X$.

The *oscillating closed orbit* is defined as that orbit which exactly repeats itself after one modulation period. The solution on turn T is obtained by putting $\hat{z} = 0$, so that

$$z_{CO} = \delta_- e^{i 2\pi Q_D T} - \delta_+ e^{-i 2\pi Q_D T} \quad (7)$$

generally following a tilted ellipse, *not* a circle, in normalized phase space. The semi-minor and semi-major axes are $||\delta_-| - |\delta_+||$ and $|\delta_-| + |\delta_+|$ long. In practice the aspect ratio of the ellipse is close to 1 when the AC dipole is driven at a tune close to the fractional betatron tune ($Q_- \approx 0$). When $|\delta_-| \gg |\delta_+|$ the approximate motion is

$$z_T \approx \hat{z} e^{i 2\pi Q_X T} + \delta_- e^{i 2\pi Q_D T} \quad (8)$$

Motion in the *rotating frame*, which rotates with the AC dipole drive at $2\pi Q_D$ per turn, is denoted by over-bars. Assuming the previous approximation to be accurate,

$$\bar{z}_T = \delta_- + \hat{z} e^{-i 2\pi Q_- T} \quad (9)$$

That is, a test particle slowly circulates the vector δ_- at a radius of constant length $|\hat{z}|$, as illustrated in Fig. 1.

When a bunch is considered, a distribution of \hat{z} values must be used. A smoothly distributed beam has $\langle \hat{z} \rangle = 0$ and $\langle \hat{z}^2 \rangle = 2\beta\epsilon_u$ where $\langle \rangle$ represents a bunch average, and ϵ_u is the unnormalized root mean square emittance. The center of charge motion in the non-rotating frame is just

$$\langle z_T \rangle = \delta_- e^{i 2\pi Q_D T} \quad (10)$$

* Work supported by the DoE.

Thus, the coherent bunch response to an adiabatically driven AC dipole has a constant amplitude

$$a_{COH} = |\delta_-| = \frac{1}{4} \left| \frac{\delta}{\sin(\pi Q_-)} \right| \quad (11)$$

This has been quantitatively confirmed in the AGS [7].

Any bunch tune spread (due to non-zero chromaticity or nonlinear detuning) trivially modifies the rate of advance around the δ_- vector for different particles. Less trivially, the tune spread also modifies δ_- , which is a function of Q_- (see Eqn. 6). This is not a practical problem if Q_D is sufficiently far outside the bunch tune spectrum.

2 SHEAR MOTION - DETUNING

The total *approximate* one turn difference map is

$$\Delta J = \frac{\delta}{\sqrt{\beta_D}} \sqrt{2J} \cos(2\pi Q_D t) \cos(\phi) \quad (12)$$

$$\Delta \phi = -\frac{\delta}{\sqrt{\beta_D}} \frac{1}{\sqrt{2J}} \cos(2\pi Q_D t) \sin(\phi) \quad (13)$$

$$+ 2\pi(Q_{X0} + \alpha J)$$

where ψ_0 is set to 0, and the action $J = a^2/2\beta_D$ has the dimensions of length. Detuning with action is present, proportional to αJ , since (if $\delta = 0$)

$$Q_X(J) \equiv \langle \Delta \phi \rangle / 2\pi = Q_{X0} + \alpha J \quad (14)$$

where $\langle \rangle$ represents a time average. A *one turn discrete Hamiltonian* H_1 concisely describes this motion, through

$$\begin{pmatrix} \Delta J \\ \Delta \phi \end{pmatrix} = \begin{pmatrix} -\partial H_1 / \partial \phi \\ \partial H_1 / \partial J \end{pmatrix} \quad (15)$$

Since H_1 represents a *difference* map, and not continuous *differential* motion, it is not (necessarily) even approximately a constant of the motion. In the case at hand

$$H_1 = 2\pi(Q_{X0}J + \frac{\alpha}{2}J^2) - \frac{\sqrt{2}\delta}{\sqrt{\beta_D}} J^{1/2} \cos(2\pi Q_D t) \sin(\phi) \quad (16)$$

This Hamiltonian is marred by its time dependence.

A canonical transformation to the rotating frame is achieved by applying the generating function

$$W(\bar{J}, \phi, t) = \bar{J}\phi - 2\pi Q_D t \bar{J} \quad (17)$$

New and old action-angle coordinates are related through

$$\begin{aligned} J &\equiv \partial W / \partial \phi = \bar{J} \\ \bar{\phi} &\equiv \partial W / \partial \bar{J} = \phi - 2\pi Q_D t \end{aligned} \quad (18)$$

If one turn motion is small ($Q_- \approx 0$), the transformation $\bar{H}_1 \equiv H_1 + \partial W / \partial t$ (averaged over many turns) yields

$$\bar{H}_1 = 2\pi(\frac{\alpha}{2}\bar{J}^2 - Q_- \bar{J}) - \frac{\delta}{\sqrt{2\beta_D}} \bar{J}^{1/2} \sin(\bar{\phi}) \quad (19)$$

\bar{H}_1 is independent of time, and is a good approximation to a constant of the motion. The fixed points are given by

$$\bar{\phi}_{FP} = \pm \frac{\pi}{2} \quad (20)$$

$$0 = 2\pi(\alpha \bar{J}_{FP} - Q_-) \mp \frac{\delta}{2\sqrt{2\beta_D}} \frac{1}{\bar{J}_{FP}^{1/2}}$$

In general there is either one stable fixed point, or one unstable and two stable fixed points [7]. When detuning is absent ($\alpha = 0$) the fixed point amplitude is

$$a_{FP} = \frac{1}{4\pi} \left| \frac{\delta}{Q_-} \right| \quad (21)$$

agreeing with Eqn. 11 when Q_- is small!

3 NONLINEAR MOTION IN 2-D

The action-angle time series $(J_x, \phi_x, J_y, \phi_y)_t$ is derived from turn-by-turn data $(x_1, x_2, y_1, y_2)_t$ recorded at 2 horizontal and 2 vertical BPMs. This requires the empirical adjustment of the β function ratio of each BPM pair, of the phase advance of each pair, and of the closed orbit error at every BPM, in a process which also corrects for the elliptical motion of the oscillating closed orbit [1, 3, 6].

The general 2-D one turn discrete Hamiltonian is

$$\begin{aligned} H_1 &= 2\pi Q_{X0} J_x + 2\pi Q_{Y0} J_y \\ &+ \sum_{ijkl} V_{ijkl} J_x^{i/2} J_y^{j/2} \sin(k\phi_x + l\phi_y + \phi_{ijkl}) \end{aligned} \quad (22)$$

where the appropriate set of indices $(ijkl)$ depends on the dominant nonlinearities [2]. Only in the simplest of models can V_{ijkl} and ϕ_{ijkl} be predicted analytically. The horizontal action time series is then

$$J_x(t) = J_{x0} - \sum_{ijkl} \frac{k V_{ijkl} J_{x0}^{i/2} J_{y0}^{j/2}}{2 \sin[\pi Q_{kl}]} \sin(2\pi Q_{kl} t + \phi_{0ijkl}) \quad (23)$$

A single harmonic dominates if $Q_{kl} = kQ_X + lQ_Y$ approaches an integer for some (k, l) pair. With coherent bunch motion, Q_X and Q_Y represent the drive tunes of simultaneous horizontal and vertical AC dipoles, and J_{x0} and J_{y0} represent the average (fixed point) actions.

The Discrete Fourier Transform (DFT) of a long action time series generates *action harmonic coefficients*

$$D_{xkl} = \frac{-k}{2 \sin[\pi Q_{kl}]} \sum_{ij} V_{ijkl} J_{x0}^{i/2} J_{y0}^{j/2} e^{i\phi_{0ijkl}} \quad (24)$$

The value of a coefficient (D_{xkl} or D_{ykl}) depends on the J_{x0} and J_{y0} values used in that measurement. Multiple measurements on a grid of (J_{x0}, J_{y0}) values can be used to recover a complete set of V_{ijkl} and ϕ_{0ijkl} values.

Sometimes the motion is summarized by *smear* statistics [2]. The horizontal smear s_{xx} is given by

$$s_{xx}^2 \equiv \frac{\langle a_x a_x \rangle}{\langle a_x \rangle \langle a_x \rangle} - 1 = \sum_{ijkl} \frac{k^2 V_{ijkl}^2 a_{x0}^{2i-4} a_{y0}^{2j}}{2^{i+j+3} \sin^2[\pi Q_{kl}]} \quad (25)$$

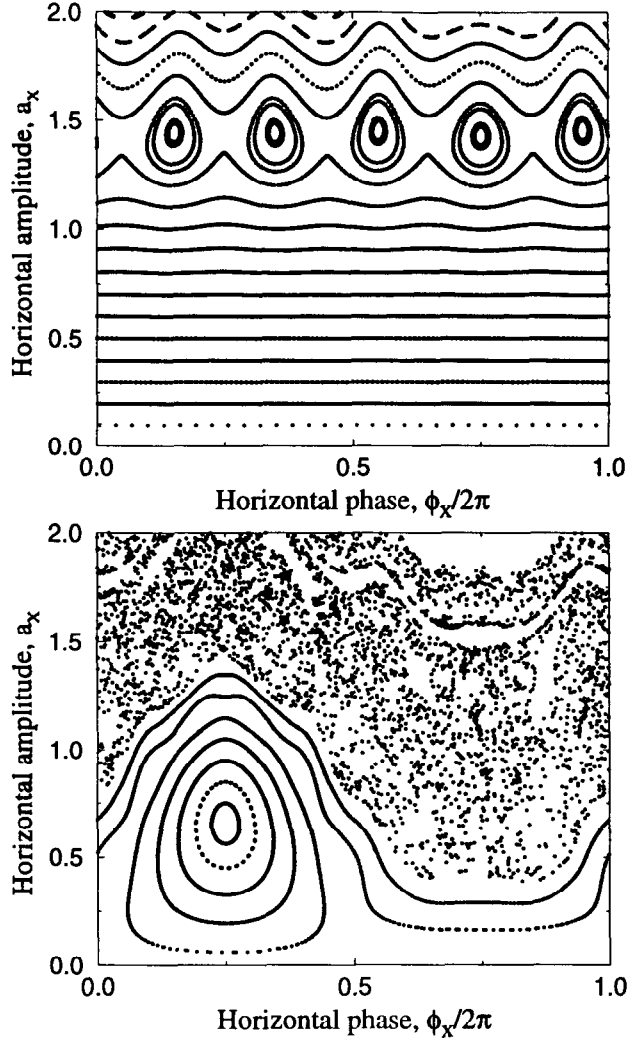


Figure 2: 1-D motion near a decapole driven resonance with an AC dipole OFF (TOP) or ON (BOTTOM).

(assuming $\beta_x = \beta_y = 1$). Similar expressions exist for the vertical and correlation smears, s_{yy} and s_{xy} . Predicted and measured smears agree well in controlled experiments with a small number of dominant nonlinearities [3, 4].

4 A NUMERICAL EXPERIMENT

Figure 2 illustrates a simple 1-D numerical experiment. Detuning (from $Q_x = 0.591$ to 0.609 between $a_x = 0$ and 2.0) is driven by three octupoles, arranged to minimize octupole driven resonances [6]. A single decapole drives the $Q_x = 3/5$ resonance, generating a chain of 5 islands at $a_{RES} \approx 1.4$ in the TOP figure. The AC dipole tune $Q_D = 59/101 \approx 0.584$ allows a plotting period of 101 turns to be used in the BOTTOM figure. A coherent AC dipole ON signal is simulated by launching a single particle at the fixed point in the BOTTOM figure, to generate a turn-by-turn “BPM” time series. The goal is to show that this time series closely resembles that obtained with the same launch coordinates, but with the AC dipole OFF.

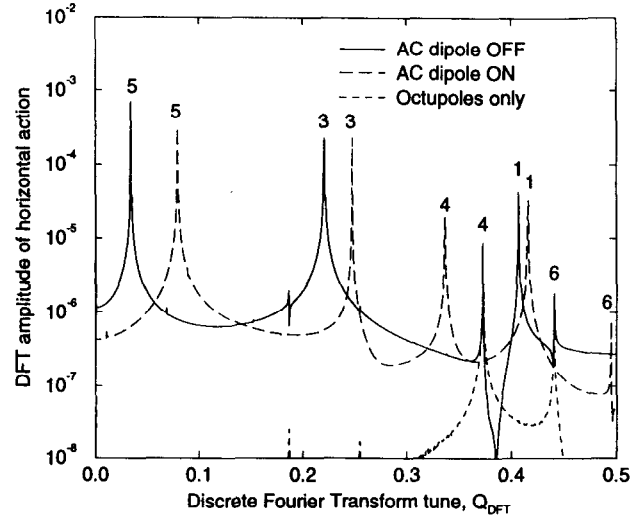


Figure 3: Discrete Fourier Transforms of the horizontal action of a test particle in the numerical experiment.

Figure 3 shows action DFTs with the AC dipole OFF and ON. The harmonic peaks shift because $Q_x \approx 0.593(0.584)$ in the OFF (ON) case. Nonetheless, the D_{x50} and D_{x30} values derived from both data sets are closely consistent, after correcting for the $1/\sin[\pi Q_{k0}]$ dependence in Eqn. 24. This implies that single particle Hamiltonian values V_{ijkl} and ϕ_{0ijkl} can indeed be derived from measurements of coherent motion driven by AC dipoles.

5 ACKNOWLEDGMENTS

Many thanks to Mei Bai, Rhianna Bianco, Wolfram Fischer, Todd Satogata, and Chunmei Tang.

6 REFERENCES

- [1] A. Chao et al, PRL 61 (1987) 2752; T. Chen et al, PRL 68 (1992) 33; T. Satogata et al, PRL 68 (1992) 1838
- [2] S. Peggs, Proc. 2nd ICFA workshop, CERN 88-04, and SSC-175 (1988)
- [3] N. Merminga, Ph.D. Thesis, U. Michigan (1989)
- [4] M.Y. Li, Ph.D. Thesis, U. Houston (1990)
- [5] S.Y. Lee et al, PRL 67 (1991) 3768; D.D. Caussyn et al, PRA 46 (1992) 7942; W. Fischer, Dissertation, U. Hamburg (1995); W. Fischer et al, PRE 55 (1997) 3507
- [6] T. Satogata, Ph.D. Thesis, Northwestern U. (1993)
- [7] M. Bai et al, PRE 56 (1997) 6002; M. Bai, Ph.D. Thesis, U. Indiana (1999); M. Bai et al, Beam manipulation with an RF dipole, these proceedings (1999)
- [8] B. Parker et al, Design of an AC dipole for use in RHIC, these proceedings (1999)
- [9] P. Castro-Garcia, Doctoral Thesis, U. de Valencia (1996)
- [10] S. Peggs and C. Tang, RHIC/AP/159, Upton, (1998)